5. Can a parallelogram with a 100° angle be inscribed in a circle?

No! Any parallelogram inscribed in a circle must be a rectangle (Theorem 94).

7.

a. If a rhombus is inscribed in a circle, what must be true about the rhombus?

It must be a square (the only rectangle that is also rhombus)

b. If a trapezoid is inscribed in a circle, what must be true about the trapezoid?

It must be an isosceles trapezoid (opposite angles must be supplementary)

8. Prove that the bisector of an angle of an inscribed triangle also bisects the arc cut off by the opposite side.

**Given:** \( \triangle ABC \) is inscribed in \( \odot O \)
BF bisects \( \angle ABC \)

**Prove:** BF bisects AC

**Statements** | **Reasons**
---|---
1. \( \triangle ABC \) is inscribed in \( \odot O \) | 1. Given
2. BF bisects \( \angle ABC \) | 2. Given
3. \( \angle ABF \equiv \angle CBF \) | 3. Definition of \( \angle \) bisector
4. AF \( \equiv \) CF | 4. \( \equiv \) inscribed \( \angle \)s intercept \( \equiv \) arcs
5. BF bisects AC | 5. Definition of arc bisector
9.

Find

a. $m_\angle ADC$

$m_\angle ADC = 180 - 115 = 65^\circ$

b. $m_\angle CDF$

$m_\angle CDF = 180 - 65 - \frac{1}{2}(60) = 85^\circ$

c. $m_\angle C$

$m_\angle C = 85^\circ$ (PAI)

d. $m_\angle A$

$m_\angle A = 95^\circ$ (Supp with $\angle C$)

10.

Find $PS$

$RP = 25$ (3-4-5 $\triangle$) and $\angle S$ is right
(supp with $\angle Q$)

$\Rightarrow PS = 24$ (7-24-25 $\triangle$)
11. Trapezoid WXYZ is circumscribed about \( O \). \( \angle X \) & \( \angle Y \) are right \( \angle \)s, \( XW = 16 \), and \( YZ = 7 \). Find the perimeter of WXYZ.

\[
P = (x + y) + 16 + (16 - x + 7 - y) + 7
\]

\[= 46\]

15. Find \( m \angle Q \)

\[
(100 - 2x) + x^2 = 180
\]

\[\Rightarrow x^2 - 2x - 80 = 0\]

\[\Rightarrow (x - 10)(x + 8) = 0\]

\[\Rightarrow x = 10 \text{ or } x = -8\]

\[\Rightarrow m \angle Q = (100 - 20) = 80^\circ \]

or

\[m \angle Q = (100 + 16) = 116^\circ \]
16. EFGH is a parallelogram with \( JO = 6 \) and \( m\widehat{HG} = 120^\circ \). Find the perimeter of EFGH.

\[ \begin{align*}
\text{Since EFGH is an inscribed parallelogram, it is a rectangle.} \\
\Rightarrow m\angle F &= 90^\circ \\
\Rightarrow m\angle GJE &= 180^\circ \\
\Rightarrow m\angle HE &= m\angle GF = 60^\circ \\
\Rightarrow m\angle GEF &= \frac{1}{2} (60) = 30^\circ \\
\text{Now, } JO = OG = OE = 6 \\
\Rightarrow GF &= 6 \text{ and } EF = 6\sqrt{3} \text{ (sides of 30-60-90 } \triangle) \\
\therefore P_{EFGH} &= 6 + 6\sqrt{3} + 6\sqrt{3} = 12 + 12\sqrt{3}
\end{align*} \]

19. Equilateral \( \triangle PQR \) is inscribed in one circle and circumscribed about another circle. The circles are concentric.

a. If the radius of the smaller \( \odot \) is 10, find the radius of the larger circle.

\( \text{Radius of larger } \odot = 20 \) (hypotenuse of 30-60-90 \( \triangle \))

b. In general, for an equilateral \( \triangle \), what is the ratio of the radius of the inscribed \( \odot \) to the radius of the circumscribed \( \odot \).

\[ \frac{\text{Radius of inscribed } \odot}{\text{Radius of circumscribed } \odot} = \frac{x}{2x} = \frac{1}{2} \]
20. 

ABCD is a kite with $AB = BC$, $AD = CD$, and $m \angle B = 120^\circ$. The radius of the circle is 3. Find the perimeter of ABCD.

23. 

Are the vertices of each figure concyclic (e.g., lie on the same circle) Always, Sometimes, or Never?

a. Rectangle

b. Parallelogram

c. Rhombus

d. Nonisosceles trapezoid

e. Equilateral polygon

f. Equiangular polygon
24. 
A right Δ has legs measuring 5 and 12. Find the ratio of the area of the inscribed ⊙ to the area of the circumscribed ⊙.

\[
(12 - x) + (5 - x) = 13 \\
\Rightarrow x = 2 \\
\Rightarrow \text{Area of inscribed } \odot = \pi(2)^2 = 4\pi
\]

\[
\text{Radius of circumscribed } \odot = \frac{13}{2}
\]

\[
\Rightarrow \text{Area of circumscribed } \odot = \left(\frac{13}{2}\right)^2 \pi = \frac{169}{4}\pi
\]

\[
\therefore \frac{\text{Area of inscribed } \odot}{\text{Area of circumscribed } \odot} = \frac{4\pi}{\frac{169}{4}\pi} = \frac{16}{169}
\]

26. 
A circle is inscribed in a triangle with sides 8, 10, and 12. The point of tangency of the 8-unit side divides that side in the ratio \(x:y\) where \(x < y\). Find that ratio.

\[
8 - x + 10 - x = 12 \\
\Rightarrow x = 3 \& 8 - x = 5
\]

\[
\therefore \text{the ratio is } \frac{3}{5}
\]